# SMALL –SIGNAL FIELD ANALYSIS OF GYRO-TWT AMPLIFIER

*Synopsis submitted in fulfillment of the requirements for the Degree of*

## DOCTOR OF PHILOSOPHY

By

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# DECLARATION BY THE SCHOLAR

I hereby declare that the work reported in the Ph.D. thesis entitled **"Small-signal Field Analysis of Gyro-TWT Amplifier"** submitted at **Jaypee Institute of Information Technology, Noida, India,** is an authentic record of my work carried out under the supervision of Prof. N. Kalyanasundaram. I have not submitted this work elsewhere for any other degree or diploma. I am fully responsible for the contents of my Ph.D. Thesis.

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# SUPERVISOR'S CERTIFICATE

This is to certify that the work reported in the Ph.D. thesis entitled "**Small-signal Field Analysis of Gyro-TWT Amplifier"**, submitted by **Jasmine Saini** at **Jaypee Institute of Information Technology, Noida**, **India** is a bonafide record of her original work carried out under my supervision. This work has not been submitted elsewhere for any other degree or diploma.

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Date: 26th August, 2013

# SYNOPSIS

The synopsis being put forward is based on my research work "*Small signal Field analysis of gyro-TWT Amplifier*".

## Gyro-TWTs

Gyro-TWT as an amplifier is widely used in radars, linear accelerators and high information density communication systems. Because of its high power and broad bandwidth capabilities, gyro-TWTs find a number of applications at millimetre and sub-millimeter wave frequencies. The interaction structure used in Gyro-TWTs is a circular cylindrical wave-guide. The gyro-TWT is a fast wave device which is based on cyclotron resonance.

## Mechanism of Small signal Amplification

The mechanism of small-signal amplification in gyro-TWTs and cyclotron resonance masers (CRMs) was actively studied by many researchers in the 1980's culminating in the derivation of the dispersion relation in the form of an infinite series. The celebrated (Doppler-shifted) cyclotron resonance condition

$$\omega - v_z \beta_{mn}(\omega) - s\,\Omega_e/\gamma = 0$$

was identified as a sufficient requirement for small-signal amplification. In the above equation, ω is the operating (radian) frequency, $\beta_{mn}(\omega)$ is the unperturbed propagation phase constant of the mn$^{th}$ waveguide mode, $v_z$ is the axial speed of the electrons, $\Omega_e$ is the electron cyclotron frequency, s is the harmonic number and γ is the relativistic factor.

In fast wave devices, the phase speed of the electromagnetic wave is greater than the speed of light *c*, and the electromagnetic wave gets amplified as a result of interaction with a beam of relativistic electrons. The electrons in the beam will rotate with an angular frequency given by $\Omega_e$ in the absence of any r.f. perturbation. In the presence of a r.f. transverse electric field, the electrons experience an additional force which causes some electrons to accelerate and others to decelerate depending on the relative phase of the electric field. Since the cyclotron frequency is inversely proportional to the relativistic mass factor, the frequency will decrease for accelerated electrons and increase for decelerated electrons. After few cycles, the electrons that gained energy lag in phase and the electrons that loose energy advance in phase, resulting in phase bunching. If the electric-field frequency is exactly equal to the electron cyclotron frequency, this bunching process will continue. This is accomplished by a slight detuning of the axial magnetic field so that the cyclotron frequency is slightly lower than the RF frequency. When this condition is achieved, then the bunches will revolve in phase with the electric field and give up rotational energy to the RF field.

In small signal amplification, the linear interaction takes place over a small distance/length where the effect of the electron beam on the field is dominant and the effect of the field on the beam is insignificant.

## Approach to the Problem

We have developed a small-signal kinetic theory of TE and TM mode interaction expressing the current and charge densities which are the sources of electromagnetic fields in terms of the electron phase-space density function. Making use of linearized Vlasov equation satisfied by the perturbed phase space density, we have derived the dispersion equation.

**The Vlasov Equation**

In order to effectively design a gyro-TWT, its performance must be predicted by an appropriate theoretical model. A theory is required that describes how the r.f. field interacts with an electron beam. The kinetic theory within the framework of nonneutral plasma offers one of the best approaches. As the (nearly) helical trajectories described by the electrons in a gyro-TWT or CRM amplifier cross one another even under small-signal conditions, the electron velocity  ceases to be a single-valued (vector) function of the current position in the Eulerian picture. Hence, recourse has to be made either to Lagrangian description wherein the trajectory of each individual electron is indexed by its initial position coordinates and the initial time (that is, at the entrance to the interaction region) or to a kinetic description within the framework of collisionless nonneutral plasma theory. Since the charged particles of a single species, namely electrons, making up the nonneutral plasma in a gyro-TWT give rise to a tenuous beam, the nonneutral plasma may be modelled to be collisionless with negligible error.

The tenuous electron beam in a typical gyro-TWT amplifier consists of a huge number of charged particles interacting with one another due to their self-fields and with any external fields that may also be present. The physical state of a particle in this collection is completely described by its position and its momentum at any time $t$, that is, in terms of their trajectories in the six dimensional space spanned by the three dimensional position vector $\boldsymbol{r}$ and the three dimensional momentum vector $\boldsymbol{p}$– the so-called phase space. In order to determine the plasma dynamics, we could, in principle, establish an equation of motion for each electron, then try to solve the resulting set of a huge number of coupled equations. Clearly, such a scheme is not feasible, and therefore, kinetic theory attempts to describe the state of the nonneutral plasma through a phase-space density $f(\boldsymbol{r}, \boldsymbol{p}, t)$. A kinetic description based on the linearized Vlasov equation turns out to be ideally suited for an analysis of small-signal amplification in gyro-TWTs.

**Dispersion Relation**

When the cyclotron resonance condition is approximately satisfied by a particular waveguide mode, the dispersion relation may be reduced to a biquadratic equation. The algebraic equation is solved for the propagation constant as a function of the operating frequency. From the root of the biquadratic equation, we can analyze the behavior of the r.f. wave.

## Analysis of TE mode interaction

In case of TE mode, single axial profile function suffices for a complete description of the TE mode field configuration. The axial profile function satisfies a second order ordinary differential equation with constant coefficients. In view of the form of the differential equations satisfied by axial profile function, a complex exponential solution is assumed. The solvability condition for a nontrivial solution yields a biquadratic algebraic equation. This biquadratic equation is the desired dispersion equation for determining the perturbed propagation phase constant.

The small-signal theory developed is used in the preliminary design of a typical gyro-TWT amplifier, whose operating frequency is 94 GHz, with beam current of 10A in a waveguide of radius 0.54 cm ,operating in various modes .

When the cyclotron-resonance condition is enforced over the entire frequency range by allowing the gyro-radius and the relativistic factor to be frequency-dependent, it was observed from the frequency-response curve that a gyro-TWT amplifier optimally designed for interaction with a TE mode of a circular cylindrical waveguide is capable of small-signal gain over a band of frequencies around the design frequency.

However, in practical gyro-TWT amplifiers, we cannot vary the amplifier parameters with respect to the frequency of the input signal. So, it is therefore, mandatory to keep the values of the relativistic factor and gyro-radius fixed. Once the amplifier parameters are fixed, the growth-rate curve no longer exhibits a peak around the design frequency but rather a monotonic variation with respect to frequency.

It may therefore be concluded that TE mode interaction is not suitable for the design of small signal gyro-TWT amplifier.

## Analysis of TM mode interaction

In case of TM mode, two axial profile functions are required for the complete description of the TM-mode field configuration. Using these axial profile functions, we have got a coupled system of two first-order ordinary differential equations with constant coefficients. In view of the form of the differential equations satisfied by axial profile function, a complex exponential solution is assumed. The solvability condition for a nontrivial solution yields a biquadratic algebraic equation. This biquadratic equation is the desired dispersion equation for determining the perturbed propagation phase constant.

The small-signal theory developed is being used in the preliminary design of a typical gyro-TWT amplifier, whose operating frequency is 94 GHz, with beam current of 10A in a waveguide of radius 0.54 cm, operating in various modes .

When the cyclotron-resonance condition is enforced over the entire frequency range by allowing the gyro-radius and the relativistic factor to be frequency-dependent, the initial growth rate of the interacting TM wave is seen to exhibit a shallow minimum close to, but shifted to the right of the design frequency. The growth–rate curve, exhibits two maxima on either side of the minimum. The maximum on the left is sharp and large, whereas, the maximum on the right is broad and small compared to the left.

As in the case of TE-mode interaction, we have to keep the amplifier parameters fixed. In practical gyro-TWT amplifiers, on fixing the parameters at appropriate frequency, the growth rate curve can be made to exhibit broad maximum at the design frequency, unlike the case of TE-mode interaction.

The interaction with a TM mode of a circular cylindrical waveguide is thus, capable of a decent small-signal gain over a broad band of frequencies around the design frequency. We have also plotted few other performance curves.

## Drawbacks and corrections in previous works

I. The small-signal theory of the linear interaction between TE and TM modes of a circular cylindrical wave-guide and beam of gyrating electrons in a gyro-TWT configuration was studied by many researchers [3,9-12]; however, every one of the reported derivations of the dispersion relation governing small-signal amplification without exception has failed to make use of the correct form of the electron phase-space density function in the kinetic-theory based approach adopted by them, thereby, ending up with erroneous results for both TE and TM mode interaction.

   We have developed a small-signal kinetic theory of TE and TM-mode interaction making use of the correct form for the equilibrium phase-space density function. The axially symmetric equilibrium distribution function corresponding to an applied axially directed uniform magnetic field turns out to be independent of $r_o$ (guiding center radius). As a consequence, the derivation of the correct dispersion relation requires integration with respect to the momentum variables $p_t$(transverse momentum) and $p_z$(axial momentum) only. This is reported in our paper "On Small Signal Amplification in a Gyro-TWT", Progress in Electromagnetic Research *C*, vol. 21, pp. 75-86, 2011.

II. There is also a common fundamental error specific to TM-mode interaction made in all of the kinetic-theory based approaches reported so far in the literature. The starting point of the kinetic-theory based linear analyses of the TM-mode interaction presented in previous works is inconsistent with Maxwell's equations. It is well known that the correct form of Maxwell's equations for magnetic field can be expressed as

$$\mathbf{\nabla_t} \times \mathbf{B}_t = (j\omega/c^2)E_z\hat{\mathbf{z}} + \mu_o J_z\hat{\mathbf{z}}$$

$$\hat{\mathbf{z}} \times \partial\mathbf{B}_t/\partial z = (j\omega/c^2)\mathbf{E}_t + \mu_o\mathbf{J}_t$$

If the $J_z$ and $\mathbf{J}_t$ on the right hand side of the above equations are set arbitrarily to zero as has been done by all the previous researchers without any justification, the homogenized versions of the above equations immediately imply the non-existent relations

$$C_{mn}(z) = -j(k_{mn}^2\, c^2/\omega)B_{mn}^{(e)}(z)$$

$$A_{mn}^{(e)}(z) = j(c^2/\omega)\, dB_{mn}^{(e)}(z)/dz$$

among the axial profile functions appearing in the literature on TM-mode interaction. In our second paper "On small-signal amplification of a TM circular cylindrical wave-guide mode in a Gyro-TWT, IET Microwaves, Antennas & Propagation, vol.7, pp.644-655, 2013", we have given a detailed analysis of small signal theory using the correct relations.

## Conclusion

The gyro-TWT is capable of working as a small signal amplifier only in TM mode and not in TE mode.

## Future Scope

The below listed points highlight some of the areas, in which this research can be extended.

1. The analysis on a large signal field theory of a gyro-TWT amplifier incorporating space-charge effects can be taken as a complete research problem.

2. We have not considered the velocity spread, $v_{t0}$ (transverse velocity) and $v_{z0}$(axial velocity) of electron beam. Considering them, one can formulate another research problem.

3. In the research work, we have considered a smooth circular cylindrical waveguide. In practice, the waveguide structure is not uniform. This work can be extended to non-uniform structure.

4. The perturbation of the electron beam arising out of the interaction is disregarded in the small-signal theory, so we cannot discuss performance indices like power gain, efficiency, optimum interaction length etc. These can be analyzed using a large signal field theory.

## Organization of the Thesis

The thesis is organized in the following way. Chapter1 introduces the importance of microwave technologies and usefulness in consumer, military, scientific and industrial applications. It gives the brief introduction of microwave tubes, comparison of slow wave and fast wave tubes, position of gyro devices in today's world. This chapter also features a comparison of conventional microwave tubes (such as Linear Beam and Crossed Field tubes) with fast wave tubes. It is followed by a discussion of the basic structure of TWT, its components and working principle. Towards the end of this chapter, a brief introduction about Gyro-TWTs is given.

Chapter 2 includes a discussion about various types of gyro-devices. Firstly, Gyrotron structure is discussed in detail. Electron beam generation, propagation, cyclotron resonance condition, electron bunching, spent beam collection, RF power generation and output extraction have been discussed in detail. Some practical gyrotron configurations are also discussed. Comparison of Gyrotrons with conventional Microwave tubes has been attempted. Other Category of gyro-devices viz., Gyro-amplifiers: Gyro-TWTs, Gyro-Klystrons and Gyro-Twystrons along with their structures have been discussed .The bunching mechanism of Gyro-TWTs has been explained in detail.

Chapter 3 includes a literature survey on the works carried out by previous researchers. This chapter extensively discusses the drawbacks of the existing derivations. This chapter discusses how the earlier derivations of the dispersion relation governing small-signal amplification without exception, have failed to make use of the correct form of the electron phase-space density function in the kinetic-theory based approach. It also discusses the common fundamental error specific to TM mode interaction made in all of the kinetic theory based approaches reported so far in the literature

Chapter 4 is devoted to a detailed analysis of the space time evolution of electron phase space density function. It also discusses the linearized Vlasov equation which is very suitable for carrying out the small-signal analysis. We have introduced the basic geometry of gyro-TWT interaction. In order to bring out the functional dependence of the axially symmetric equilibrium phase-space density on the position and the momentum variables, the single-particle constants of motion have been identified. Proof of the independence of the equilibrium phase-space density function on guiding center radius is given at the end of this Chapter.

In Chapter 5, the small signal amplification of a TE circular cylindrical wave-guide mode linearly interacting with an annular beam of gyrating electrons in a gyro-TWT configuration is analyzed. The axial profile functions which are required for the complete description of the TE mode field configuration are identified. Using the linearized Vlasov equation satisfied by the phase–space density function within the framework of nonneutral plasma kinetic theory, we have arrived at the dispersion equation. TE mode discussions are carried out towards the end of the Chapter 5.

In Chapter 6, the small signal amplification of a TM circular cylindrical wave-guide mode linearly interacting with an annular beam of gyrating electrons in a gyro-TWT configuration is analyzed. With the help of these axial profile functions that are required for the complete description of the TM mode interaction and using the linearized Vlasov equation satisfied by the phase–space density function within the framework of nonneutral plasma kinetic theory, we have arrived at the dispersion equation.

In Chapter 7, the small-signal theory developed in Chapters 5 and 6 is used in the preliminary design of a typical gyro-TWT amplifier, whose operating frequency is 94 GHz, with a beam current of 10A in a waveguide of radius 0.54cm, operating in various modes .

Possible directions in which the present work can be extended are discussed in the concluding Chapter 8.

## Publications

1. **Kalyanasundaram, N., and Jasmine Saini, "**On small-signal amplification of a TM circular cylindrical wave-guide mode in a Gyro-TWT", IET Microwaves, Antennas & Propagation, vol.7, pp.644-655, 2013 [Impact factor 0.681]

2. **Kalyanasundaram, N.**, and **Saini J.**, "On Small Signal Amplification in a Gyro-TWT", Progress in Electromagnetic Research *C*, vol. 21, pp. 75-86, 2011 [Impact factor 4.735]

3. **N.Kalyanasundaram**, **Jasmine Saini, G.Naveen Babu**, "Small-signal Field Analysis of Gyro-TWT Amplifier", IEEE International Conference on Vacuum Tubes (IVEC-2011), Bangalore, India, pp. 337-338, Feb 2011. [Indexed in Scopus]

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