

# **SOME THERMAL STABILITY PROBLEMS OF EASTICO-VISCOUS, FERROMAGNETIC AND NANOFUIDS**

*Synopsis of the thesis submitted in fulfillment of the requirements for the  
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## INTRODUCTION

The science which deals with the properties of fluids in motion is called fluid dynamics. It is one of the most important areas of physical sciences and has a wide range of applications, including calculating forces and moments on aircrafts, determining the mass flow rate of petroleum through pipelines, predicting weather patterns. Moreover, fluid dynamics has numerous and important applications to engineering, geophysics, astrophysics, aerodynamics, meteorology, oceanography, plasma physics, pump and turbines, ground water seepage and the extraction of oil from underground reservoirs etc.

In fluid dynamics, we make use of continuum theory though we know that all matter is composed of a large number of molecules, which are in random motion. Fluid has molecules bombarding each other. Matter is therefore discontinuous or discrete at macroscopic scales. In principle, it is possible to study the mechanics of a fluid by studying the motion of the molecules themselves, as is done in kinetic theory or statistical mechanics. However, we are generally interested in gross behaviour of the fluid, that is, in the average manifestation of the molecular motion. For example, forces are exerted on the boundaries of a container due to the constant bombardment of the molecules; the statistical average of this force per unit area is called pressure, a macroscopic pressure; so long as we are not interested in the mechanism of the origin of pressure and think of pressure as simply “force per unit area”. It is thus possible to ignore the discrete molecular structure of matter and replace it by a continuous distribution, called a continuum. This is called “continuum hypothesis”. Further, we shall restrict ourselves to those systems where particle velocities are small compared with the velocity of light, so that relativistic effects are not prominent.

Stability is the ability of an object to maintain equilibrium or resume its original, upright position after displacement. To investigate the stability of a system, it is subjected to arbitrary small perturbations and the reaction of the system to such an arbitrary small perturbation is studied. For an equilibrium state or a steady flow to be of permanent type, it must not only satisfy the mechanical equations but must also be stable against arbitrary small perturbations. We consider a hydrodynamic or hydromagnetic system which in accordance with the equations governing it, is in a stationary state i.e. in a state in which none of the variables describing it is a function of time. If the disturbances gradually die down or the system never departs appreciably from this stationary state, the system is said to be stable. If the system departs more and more from the initial state and never reverts to

it, the system is said to be unstable. If the system neither tends to return to its initial state nor departs from its initial position after a small displacement, we say that the system is in neutral equilibrium.

Thermal instability theory (or Bénard convection) has attracted considerable interest and has been recognized as a problem of fundamental importance in many areas of fluid dynamics. The earliest experiments to demonstrate the onset of thermal instability in fluids are attributed to Bénard [1]. In the majority of situations, if a layer of fluid is heated from below, the fluid in the lower part of the layer expands as it becomes hotter. Thus, on the account of thermal convection, the fluid at the bottom will be lighter than the fluid at the top. This is a top-heavy arrangement, which is potentially unstable and when the temperature gradient or layer depth is sufficiently large to overcome the effect of gravity, the fluid rises and a pattern of cellular motion may be seen. This is called Bénard convection (Bénard [1]).

In the standard Bénard problem, the instability is driven by a density difference which is caused by a temperature difference between the upper and lower planes bounding the fluid. If the fluid layer additionally has salt dissolved in it, then there are potentially two destabilizing sources for the density difference i.e. the temperature field and the salt field. When the simultaneous presence of two or more components with different diffusivities is considered, the phenomenon of convection which arises is called thermosolutal or double-diffusive convection. The problem of thermosolutal convection in a layer of fluid heated from below has been studied by Aggarwal and Prakash [2].

A good review of thermal stabilities has been given in the celebrated monograph by Chandrasekhar [3]. The linearized stability theory and normal mode analysis are used to determine the critical values of Rayleigh number, demarcating a region of stability from instability. Linear theory gives the conditions under which hydrodynamic systems are definitely unstable. Normal mode analysis is quite general and it gives complete information about instability, including the rate of growth of any unstable perturbation.

In the study of problems of thermal convection, it is a frequent practice to simplify the basic equations by introducing certain approximations which are attributed to Boussinesq [4]. The Boussinesq approximation can best be summarized by the following two statements:

- (i) The fluctuations in density which appear with the advent of motion result principally from thermal (as opposed to pressure) effects.

- (ii) In the equations for the rate of change of momentum and mass, density variations may be neglected except when they are coupled to the gravitational acceleration in the buoyancy force.

The formulation and derivation of the basic equations of a layer of a fluid heated from below in porous medium, using Boussinesq approximation, has been given in a treatise by Joseph [5].

There are many elasto-viscous fluids that cannot be characterized by the constitutive relations. Two such classes of fluids are Rivlin-Ericksen and Walters' (model B') fluid. Walters' [6] has proposed the constitutive equations for such elasto-viscous fluids. The mixture of polymethyl methacrylate and pyridine at 25°C containing 30.5 gm of polymer per litre behaves very nearly as the Walters' (model B') elasto-viscous fluid. Thermal stability of an elasto-viscous fluid has been studied by Bhatia and Steiner [7]. Rivlin and Ericksen [8] have proposed a theoretical model of such another elasto-viscous fluid and other polymers are used in agriculture, communication appliances and in biomedical applications. The thermosolubility stability of Rivlin-Ericksen fluid has been studied by many authors [9-11].

Stokes [12] has formulated the theory of couple-stress fluid. One of the applications of couple stress fluid is its use in the study of the mechanisms of lubrication of synovial fluids, which has become the objective of scientific research. The presence of small amounts of additives in a lubricant can improve bearing performance by increasing the lubricant viscosity and thus producing an increase in the load capacity. These additives in a lubricant also reduce the coefficient of friction and increase the temperature range in which the bearing can operate. The synovial fluid has been modeled as a couple-stress fluid in human joints by Walicki and Walicka [13]. The couple-stress fluid heated from below in presence of magnetic field and rotation has been considered by Sunil et al. [14].

Ferromagnetic fluids are electrically non-conducting colloidal suspensions of solid ferromagnetic particles in non-electrically conducting carrier fluids like water, kerosene, hydrocarbon etc. A typical ferromagnetic fluid contains  $10^{23}$  particles per cubic meter. These fluids behave as a homogeneous continuum and exhibit a variety of interesting phenomena. Ferromagnetic fluids are not found in nature but are artificially synthesized. The method of formation of ferrofluids evolved in the early to mid 1960's. An authoritative introduction to ferrohydrodynamics has been discussed in detail in the celebrated monograph by Rosensweig [15]. This monograph reviews several applications

of heat transfer through ferrofluids. In the last millennium, the investigation on ferrofluids attracted researchers because of the increase of applications in areas such as instrumentation, lubrication, vacuum technology, vibration damping, pressure seals for compressors and blowers, chemical reactor and high speed silent printers etc. Experimental and theoretical physicists and engineers have significantly contributed in the areas of ferrohydrodynamics (FHD) and its applications [16]. The theory of convective instability of ferromagnetic fluids began with Finlayson [17] and also considered by several authors [18-21].

Nanofluids are mixtures of a regular fluid, with a very small amount of suspended metallic or metallic oxide nanoparticles or nanotubes, which is first utilized by Choi [22]. Nanoparticle materials may be taken as oxide ceramics ( $\text{Al}_2\text{O}_3$ , CuO), metal carbides (SiC), nitrides (AlN, SiN) or metals (Al, Cu) etc. Base fluids are water, ethylene or tri-ethylene glycols and other coolants, oil and other lubricants, bio-fluids, polymer solutions, other common fluids. Typical dimension of the nanoparticles is in the range of a few to about 100nm. In recent years, considerable interest has been evinced in the study of Nanofluid and it has become an innovative idea for thermal engineering because it has various applications in automotive industries, energy saving, nuclear reactors etc. The general topic of heat transfer in nanofluids has been surveyed in a review article by Das and Choi [23] and a book by Das et al. [24]. The thermal instability of nanofluids in natural convection was studied by Tzou [25-26]. The onset of double diffusive convection in a nanofluid layer has been studied by Nield and Kuznetsov [27] and Kuznetsov and Nield [28].

In the literature, the effects of various parameters such as Hall currents, magnetic field, rotation, suspended particles, compressibility, porous medium, concentration Rayleigh number, solute gradient, Darcy number, Lewis number, modified diffusivity ratio etc. on the thermal stability of hydrodynamic and hydromagnetic systems is studied.

In the presence of strong electric field, the electric conductivity is found to be affected by the magnetic field. Consequently, the conductivity parallel to the electric field is reduced in the direction normal to both electric and magnetic fields. This phenomenon is known as Hall Effect. The effect of Hall currents on thermal stability has also been studied by Gupta and Aggarwal [29].

Consider a fluid to be electrically conducting and under the influence of a magnetic field. The electrical conductivity of the fluid and the prevalence of magnetic fields contribute to effects of two kinds. First, by the motion of the electrically conducting fluid

across the magnetic lines of force, electric currents are generated and the associated magnetic fields contribute to changes in the existing fields; secondly, the fact that the fluid elements carrying currents traverse magnetic lines of forces contribute to additional forces acting on the fluid elements. It is thus two-fold interaction between the motions and the fields that is responsible for patterns of behavior, which are often striking and unexpected. The interaction between the fluid motions and magnetic fields are contained in Maxwell's equations. As a consequence of Maxwell's equations, the equations of hydrodynamics are modified suitably.

The physical aspect of convection in a rotating fluid is the driving force for analysis. As convection in a rotating system is relevant to many geophysical and industrial applications such as semiconductor crystal growing, microgravity environments, etc., the theoretical and experimental analysis of these problems has extensively studied in the literature (Chandrasekhar [4, 30], Rossby [31]). Thermal convection in a rotating layer of a porous medium saturated by a homogeneous fluid is a subject of practical interest for its applications in engineering. Among the applications in engineering disciplines one can find the food process industry, chemical process industry, solidification and centrifugal casting of metals and rotating machinery. More detailed discussions of applications of thermal convection in porous medium and particularly in rotating porous domains are presented by Nield and Bejan [32].

Various studies in viscous, viscoelastic, couple-stress fluids with suspended particles (dust particles), have appeared in the literature [33-36]. The influence of suspended particles on viscoelastic flows has a great importance in petroleum industry, pulp and paper technology, purification of crude oil and several geophysical situations. The present study of suspended particles can serve as a theoretical support for experimental investigations.

Fluids are divided into two categories. Those which undergo appreciable variations in density and volume under the impressed forces fall under the category of 'compressible fluids'. Then, there are those which undergo no noticeable changes in density and volume during motion. They are termed 'incompressible fluids'. Compressibility is thus a measure of the change in density and consequently, the change in volume of a fluid under the effect of external forces. For compressible fluids, the equations governing the system become quite complicated. Spiegel and Veronis [37] have simplified the set of equations governing the flow of compressible fluids assuming that the depth of the fluid layer is much smaller than the scale height as defined by them and the motions of infinitesimal amplitude are

considered. The effect of compressibility and suspended particles on thermal stability of elastico-viscous fluids has been studied by Sharma and Aggarwal [38] and Sharma et. al [39].

A medium which is a solid body containing pores is called a porous medium. Flow through porous media is also of interest in chemical engineering (adsorption, filtration, and flow in packed columns), petroleum engineering, hydrology, soil physics, biophysics and geophysics. Two macroscopic properties of porous media which may be used to describe fluid flow are porosity and permeability.

The non-Newtonian fluids are of vital importance due to their diverse application aspects in modern technology, industries and bio-mechanics. Thus, the analysis of thermal stability and thermosolutal stability of such fluids like Rivlin-Ericksen fluids, couple-stress fluids, ferromagnetic fluids and nanofluids are desirable. We have used the two fundamental hypotheses i.e., continuum hypothesis and Newtonian mechanics throughout the study. In the present thesis, the linearized stability theory and normal mode analysis have been used to study the effects of various important parameters like suspended particles, compressibility, rotation, magnetic field, Hall currents, solute gradient, porous medium, concentration Rayleigh number, Darcy number, Lewis number and modified diffusivity ratio etc. on various stability problems of elastico-viscous, ferromagnetic and nanofluids.

## **OBJECTIVE**

The main objectives of the work reported in the thesis are:

- To study the thermal and thermosolutal stability of hydrodynamic and hydromagnetic systems.
- To study the effect of various parameters like rotation, magnetic field, suspended particles, compressibility, porous medium, Hall currents, concentration Rayleigh number, solute gradient, Darcy number, Lewis number and modified diffusivity ratio etc. on various stability problems of elastico-viscous, ferromagnetic and nanofluids.
- To find the parameters which are responsible for the oscillatory modes.

## **CONTRIBUTION**

The work embodied in the present thesis is divided into six chapters. Chapter wise summary of work done is as follows:

## **CHAPTER 1**

This Chapter is introductory in nature. It reviews existing literature relevant to the thesis e.g. hydrodynamics, magnetohydrodynamics and ferrohydrodynamics, types of fluids, stability of the system, methods determining stability etc. The thermal stability and thermosolutal stability problems have been described and effects of various parameters like Hall currents, magnetic field, rotation, compressibility, suspended particles, porous medium, concentration Rayleigh number, solute gradient, Taylor number, Darcy number, Lewis number and modified diffusivity ratio have been discussed.

## **CHAPTER 2**

Chapter two is divided into two sections. In these two sections, effect of rotation and magnetic field on thermal stability of viscoelastic fluid is studied. In first section, the effect of suspended particles on the thermal stability of Walter's (model B') viscoelastic fluid in hydromagnetics with rotation is considered. Using a linearized stability theory and normal mode analysis for a fluid layer between two free boundaries, an exact solution is obtained. A dispersion relation governing the effects of rotation, magnetic field and suspended particles has been derived. For the case of stationary convection, Walters's (model B') viscoelastic fluid behaves like a Newtonian fluid and it is found that rotation and magnetic field have stabilizing effect whereas suspended particles have destabilizing effect on the thermal stability of the system. Further, it is also observed that principle of exchange of stabilities is satisfied in the absence of magnetic field.

Second section deals with the thermal stability of a layer of compressible visco-elastic Walters' (model B') fluid in the presence of rotation and magnetic field to include the effects of suspended particles and porous medium. This section is an extension of the result obtained by Gupta and Aggarwal [29] to include the effects of magnetic field, rotation and porous medium in the absence of Hall currents. In case of stationary convection, it is found that the rotation has stabilizing effect on the system. The magnetic field may have destabilizing effect on the system in the presence of rotation while in the absence of rotation it always has stabilizing effect. The medium permeability has destabilizing effect on the system in the absence of rotation while in the presence of rotation it may have stabilizing effect. The suspended particles and compressibility always have destabilizing effect. Due to vanishing of visco-elastic parameter, the compressible visco-elastic fluid behaves like Newtonian fluid. The viscoelasticity, magnetic field and rotation are found to introduce oscillatory modes into the system which were non-existent in their absence.

### **CHAPTER 3**

It is divided into two sections. First section deals with the theoretical investigation on the effect of magnetic field on a rotating layer of a ferromagnetic fluid heated from below. This problem is an extension of the result reported by Aggarwal and Prakash [19] to include the effect of magnetic field in the absence of suspended particles. For the case of stationary convection, it is found that magnetic field, rotation and magnetization have stabilizing effect on the thermal stability of the system. The principle of exchange of stability is not valid for the problem under consideration, whereas in the absence of rotation and magnetic field, it is valid.

Section II is further extension of the section I permeated with suspended particles as additional parameter. For the case of stationary convection, it is found that suspended particles have destabilizing effect whereas rotation and magnetization have stabilizing effect on the system. Magnetic field has stabilizing effect on the system under certain conditions. The effects of various parameters on the thermal stability are depicted graphically also and the results are in agreement with analytical solutions. The principle of exchange of stabilities is found to hold true for the ferromagnetic fluid heated from below in the absence of rotation and magnetic field. The oscillatory modes are introduced due to the presence of rotation and magnetic field which was non-existent in their absence.

### **CHAPTER 4**

In this chapter, the effect of Hall currents has been studied on thermal stability of couple-stress and Rivlin-Ericksen fluid analytically. The effect of Hall currents on couple stress fluid permeated with dust particles seems to be uninvestigated so far, hence this section deals with the effect of Hall currents on dusty couple stress fluid. For the case of stationary convection, dust particles and Hall effect are found to have destabilizing effect on the system. Magnetic field and couple stress parameter are found to have stabilizing effect on the system. Stability of the system and oscillatory modes are considered and it is found that due to the presence of Hall effect (hence magnetic field), oscillatory modes are produced which were non-existent in their absence.

In second section, we have extended the results reported by Gupta and Sharma [11] to include the effect of suspended particles and porous medium in place of rotation and magnetic field. The perturbation equations are analyzed in terms of normal modes after linearizing the relevant hydromagnetic and Maxwell's equations. A dispersion relation governing the effects of viscoelasticity, solute gradient, compressibility, permeability, suspended particles, magnetic field and Hall currents is derived. For the case of stationary

convection, Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid due to the vanishing of the viscoelastic parameter. Compressibility, suspended particles and Hall currents have destabilizing effect whereas solute gradient and magnetic field are found to have stabilizing effect on the system. The permeability has destabilizing effect on the system. The solute gradient and Hall currents (hence magnetic field) introduce oscillatory modes into the system which was non-existent in their absence.

## **CHAPTER 5**

This chapter is divided into two sections. First section deals with the effect of rotation on the stability of double diffusive convection of a nanofluid layer. The work of Nield and Kuznetsov [27] is extended in this section by including the effect of rotation. The model used for the nanofluid incorporates the effect of Brownian motion and thermophoresis. The onset criterion for stationary and oscillatory convection is derived analytically and graphically. The effects of the concentration Rayleigh number, solute gradient, Taylor number, Darcy number, Lewis number, modified diffusivity ratio and porosity on the stability of the system are investigated.

The second section deals with the thermal stability of double diffusive convection of a rotating nanofluid layer in porous medium extending the work of Kuznetsov and Nield [28] by including the effect of rotation.

## **CHAPTER 6**

This chapter summarizes the entire work reported in the thesis and presents an overview of the future scope.

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## **Publications**

### **International Journals**

1. Aggarwal A. K., Verma A., “*The effect of compressibility, rotation and magnetic field on thermal stability of Walters’ fluid permeated with suspended particles in porous medium*” Thermal Science, doi: [10.2298/TSCI110805087A](https://doi.org/10.2298/TSCI110805087A), June, 2012. (Indexed in SCOPUS, Impact Factor - 1.45).
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5. Aggarwal A. K., Verma A. “*Hall effect on dusty couple stress fluid*”. (Communicated to Archives of Thermodynamics)

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1. Aggarwal A. K., Verma A., “*Effect of rotation and magnetic field on thermal stability of ferromagnetic fluid*”, Paper Number: IMECE2013-64288, Proceedings of the ASME 2013 International Mechanical Engineering Congress & Exposition (IMECE2013), San Diego, California, USA, November 15-21, 2013. (Indexed in SCOPUS).
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