SYNOPSIS

INTRODUCTION

The Traveling Wave Tube Amplifier (TWTA), with its wide bandwidth and large power output holds a prominent place among present day power amplifiers at microwave frequencies. The phenomenal growth of the satellite communication industry and the proliferation of radar applications have fuelled an unprecedented demand for Traveling Wave Tubes (TWTs). The most important features of helix TWTs are high power, broad bandwidth, high gain, high efficiency, long life, excellent reliability and robustness. In high power applications, the tube will invariably be operated on the verge of saturation and hence a theoretical analysis of its large signal behaviour will be of immense interest.

PROCESS OF LARGE SIGNAL AMPLIFICATION

The amplification of radio frequency signals in TWTA is achieved from the electron beam-wave interaction process. During this process of electron beam-wave interaction, a net transfer of the kinetic energy from the electron beam to the radio frequency wave supported by the slow-wave structure takes place, resulting in the amplification of the input signal. A slow-wave structure, in which the phase speed of the propagating radio frequency wave is smaller than the speed of light, is required in order to bring down the phase speed of the traveling electromagnetic wave close to the speed of electrons for effective interaction. When the electron beam enters the helical interaction region, it gets affected by the radio frequency wave. The axial component of the electric field of the input radio frequency wave accelerates some electrons and decelerates others, with the result the electron velocity and the electron arrival time become function of the electron’s location and its entrance time. The electron arrival time is the time at which an electron arrives at a location with an entrance time.

MODEL DESCRIPTION

The helix is geometrically simple and easy to fabricate; however the helix geometry makes the task of finding exact solution of boundary value problems for Maxwell's equations is quite difficult. For this reason, a number of simplified models for the helix have been proposed in the literature with a view of making the analysis of helical structures tractable. The prominent among such models are those of a tape helix and a sheath helix.

The actual helix ribbon used in TWTs has a finite thickness and finite width with rounded edges. Also, the helix is of finite length with finite conductivity. By letting the thicknesses of the helix ribbon (of rectangular cross section) go to zero and the conductivity of ribbon material go to infinity, we arrive at the tape-helix model. Thus, a tape helix consists of an infinitesimally thin and perfectly conducting tape (of constant width) wound into a helical structure of constant pitch p. Based on assumption that the current-flows along the surface of the tape, the modeling of tape may be classified as anisotropically conducting or perfectly conducting. In an anisotropically conducting model of the tape helix, the contribution of the tape current density component perpendicular to the winding direction is neglected. In the case of perfectly conducting model of the tape helix, it is no longer insisted that the tape helix is perfectly conducting along the winding direction only. If the spacing between the turns and the ribbon width of an anisotropically conducting tape helix are made to approach to zero, the resultant structure becomes electrically smooth. This anisotripically conducting cylindrical model of a tape helix is called the sheath helix. Thus the sheath helix is an infinitesimally thin cylinder, conducting only in the helical direction. In this research work, the sheath-helix model of the slow-wave structure is used for the analysis. Now the boundary conditions for the electric field at the boundary surface of the sheath helix may be approximated by the conditions that the conductivity along the tape winding direction is infinite, whereas as the conductivity normal to the winding direction on the tape surface is zero. The use of these boundary conditions permits a solution for the electromagnetic field guided by the helix to be obtained with relative ease.

In a practical TWT, the helix is supported by a number of dielectric rods which could typically be of a rectangular, circular or tapered cross-section and the entire assembly is enclosed in a perfectly conducting metal envelope. Two dielectric materials are widely used for supporting the helices of TWTs: Beryllium Oxide or Beryllia (BeO, ε_r = 6.3) and Anisotropic Pyrolitic Boron Nitride (APBN, ε_r = 5.2). These two materials have relatively high thermal conductivity. With the use of high thermal conductivity ceramics, the temperature drop across the support rods is not a severe problem in high-power helix TWTs.

In the present research work, the large-signal field analysis is done for a practically relevant model of a sheath-helix supported inside a coaxial perfectly cylindrical shell by symmetrically disposed wedge-shaped dielectric rods. In order to make the analysis of the TWTA tractable, the above model of the slow-wave structure is simplified further by replacing the azimuthally periodic dielectric constant of the tubular region between the helix and the outer conductor by its azimuthally averaged constant value $\varepsilon_{eff.}$ This simplified model of the slow-wave structure is referred to as 'dielectric-loaded sheath helix'.

APPROACH TO THE PROBLEM

The analysis of TWT as an amplifier has been carried out by Pierce and Kompfner [1-3]. This theory was developed some six and half decades ago, and it was based on a coupled-wave analysis utilizing the vacuum modes of the helix and the positive and the negative energy space – charge waves of the beam. An improved theory based on an eigen-vector analysis of Maxwell's equations for the helix has been developed by Reydbeck [4]. Chu and Jackson [5] and Collin [6] have considered the field approach for a small-signal analysis of the TWTA.

However all of these analyses have made use of either an oversimplified transmission-line model or a small signal field theory model notwithstanding the fact that any rigorous analysis of the traveling wave tube has to be necessarily based on the field approach applicable to large signals. Freund et al [7, 8] presented a linear field analysis for a TWTA using radial admittances at the boundaries. The analysis was based on the coupled-mode theory [7] and the linearized relativistic field theory [8]. Being linear theories, they are not applicable to large signals.

It is a well-known fact that in a TWTA, the entering electrons must be indexed by their respective entrance times and locations. An important impact on theoretical considerations is that the velocity becomes multivalued. Thus the theoretical techniques (usually referred to as Eulerian approach) that treat the electron beam as a fluid cannot be used. Instead of fluid approach, the theory must be able to follow individual particles or groups of particles. This is referred to as the Lagrangian approach to the analysis.

A large-signal field analysis of a TWTA for an open sheath-helix model for the slow wave structure was presented for the first time by Kalyanasundaram [9] resorting to such a Lagrangian description for the electron trajectories.

In this research work, the large-signal field analysis method of [9] is extended to the practically relevant dielectric loaded sheath-helix model.

ANALYSIS OF TRAVELING WAVE TUBE AMPLIFIER FOR A DIELECTRIC-LOADED SHEATH-HELIX MODEL

A field-theoretic analysis of the nonlinear electron-wave interaction taking place in a linear-beam TWTA operating in an axially-symmetric mode with an axially-confined electron beam involves the following steps:

1. Write Maxwell's equations for the field components and electron ballistic equation for the electron beam.

2. Express the current density and the charge density in terms of the electron arrival time.

3. The current density and the charge density, along with the electromagnetic field components, are periodic functions of time in the steady state. They can hence be expanded in complex exponential Fourier series in time variable *t*. Change of variable formula for a many-to-one map is used to represent the Fourier coefficients of the current and the charge densities as nonlinear functionals of the electron-arrival time.
4. Expand the field components in Fourier series. Substitute the Fourier series expansion of field components and the convection current density into the Maxwell's field equations, resulting in a nonhomogeneous system of partial differential equations for the field components.
5. The boundary conditions (the helix boundary conditions and the beam boundary conditions) and the signalling conditions are expressed in terms of the Fourier coefficients of the field components.
6. Steps 2 and 3 describe a nonhomogeneous boundary value problem. Solving this boundary value problem with the help of a second Fourier series expansion in the axial coordinate, the Green's functions for slow-wave structure were determined.
7. The axial electric field component inside the electron beam will then expressed as a nonlinear functional of the electron-arrival time through the above set of Green's functions.
8. The electron ballistic equation is integrated to obtain the electron arrival time in terms of the axial electric field.
9. Steps 7 and 8 are solved iteratively to get the converged value of the electron arrival time.
10. Once the electron arrival time is obtained, use step 7 to obtain axial electric field.
11. The expressions for the other electromagnetic field components will simultaneously be obtained.

12. Armed with complete knowledge of an electron's arrival time as a function of its entrance time and position coordinates, we have computed all the parameters of TWTA such as induced surface current density, power gain, conversion efficiency, optimum interaction length and harmonic generation etc.

NUMERICAL COMPUTATION

The numerical computation of the TWTA characteristics discussed in this work is based on the double Fourier – series representation (in the time variable and the axial coordinate) of the particular solution for the axial electric and magnetic field components truncated at the third temporal harmonic and the 64th spatial harmonic. The computations are carried out for an operating frequency f_0 of *6 GHz* and the dimensionless interaction length $d = (\omega_0 \; \bar{d} / v_0)$ varying between *120* to *160,* where $\bar{d}$ is the actual interaction length of TWT, ω_0 is the radian frequency of the radio frequency input signal and v_0 is the mean value of the axial electron speed at the entrance plane. A parameter α, defined by

$$\alpha \triangleq 10 \log_{10}(P_{in} / P_{dc})$$

is introduced for describing the numerical results of this work. In the above definition P_{in} is the input-signal power and P_{dc} is the 'dc' power of the beam.

The variation of electron arrival time and exit electron speed with entrance time is plotted and it is observed from the graphs that the electron arrival time is a monotonically increasing function of t_0 for $\alpha = -50$ dB, but not so for $\alpha = -40$ dB, *-30* dB *and -20* dB. This observation is in conformity with the well-known fact that electron overtaking is the rule if the input-signal power is sufficiently large. It is also observed from the graphs for electron arrival time that electrons entering the interaction region over a period of the input signal undergo a net retardation or acceleration to varying amounts, relative to their unperturbed trajectories, depending on their entrance time, their radial position and input-signal level. The relative number of electrons in the retarded category and the maximum phase lag are seen in general, to increase with the input-signal level.

The plots of exit electron speed shows that both the negative as well as the positive deviation of the exit electron speed from the entrance electron speed over a period of the input signal increases with input-signal level, the negative deviation tending to dominate over the positive deviation with an increase in the input signal level. Moreover, the perturbation of the exit electron speed from the entrance speed tends to be larger for the peripheral electrons than for the axial electrons.

We have also plotted the axial variation of the first two harmonics of the axial electric field component for beam axis, beam boundary and helix boundary for the parameter values $d = 120$, $\alpha = -20$ dB. It is seen from these graphs that the second harmonic field is smaller by at least two orders of magnitude than the fundamental field all through the interaction region. A comparison of these curves also reveals that the strongest interaction between the electron beam and the electromagnetic field takes place near the beam axis; the interaction becomes the weakest at the beam boundary and grows once again progressively stronger as the helix boundary is approached. Further it is observed that the radial variation of the axial electric field component is not affected to any significant extent by the axial position at which the field is evaluated.

The density of the surface current flowing along the sheath helix may be evaluated using the boundary condition on the tangential component of the magnetic field across a discontinuity surface. The variation in magnitude of the surface-current density with axial distance z is plotted for dimensionless interaction length d = 150 and α = -20 dB, which shows a near-exponential build-up of the surface-current density with interaction distance.

The variation of the fundamental power gain g_1 with the normalized interaction length d for α = -20 dB and α = -30 dB are also plotted. It is seen from these plots that the power gain attains flat maxima at about d=134 for an input signal level of

α = -20 dB and at about d=150 for α = -30 dB, the corresponding values of saturation gains being about 23 dB and 34 dB respectively. These observations lend credence to the anticipated behaviour of the optimum interaction length and the saturation power gain, namely, values of both parameters increase as the input-signal level is decreased. The variation of the fundamental power gain g_1 with input power level α for d=135 shows a monotonic decrease in power gain as the input-signal power is increased with the decay rate coming down from a large value near α = -50 dB to a very small value near α = -20 dB.

The variation of the fundamental conversion efficiency η_1 with normalized interaction length d for α = -20 dB and α = -30 dB is also plotted and it is observed from these plots that the fundamental conversion efficiency attains flat maxima a at about $d=134$ for an input signal level of $\alpha = -20$ dB and at about $d=150$ for $\alpha = -30$ dB, the corresponding values of saturation efficiencies being about 60 % and 50 % respectively.

CONCLUSION

An examination of the results drawn from the presented study lead to the following conclusions

1. The values of the saturation power gain and the efficiency are 23 dB and 60 %. These values are compared very favourably with the experimentally observed values [14].
2. There exists an optimum interaction length, which is a function of the parameter values and the input-signal level, for a linear beam TWTA as in the case of a klystron amplifier [15]. The optimum interaction length d_{opt} for a TWTA is defined as the interaction length at which the fundamental power gain and the conversion efficiency attain their flat maxima.
3. A linear beam TWTA is a very poor harmonic generator and consequently a very poor frequency converter even under large-signal conditions. Thus the harmonic distortion introduced by a linear beam TWTA with the slow-wave

structure modelled by a dielectric-loaded sheath helix is negligibly small even when operating at large signal levels.

4. The normalized interaction length of the TWTA is defined as $d = \omega_0 \bar{d} / v_0$ where $\bar{d}$ is the actual interaction length of TWTA, ω_0 is the radian frequency of the radio frequency input signal and v_0 is the mean value of the axial electron speed at the entrance plane. Since for a TWT $\bar{d}$ and v_0 are fixed hence a varying d corresponds to the variation in ω_0. Hence the variation of power gain with normalized interaction length will also give the frequency response of the power gain. Thus the power gain vs. normalized interaction length curves may be interpreted as frequency-response curves of the power gain for a TWTA (with a fixed interaction length of course). For a TWTA designed for operation around a centre frequency f_c corresponding to the optimum interaction length d_{opt} for a specified value of the peak gain, the flatness of the maximum in the frequency curve of TWTA implies that the TWTA is capable of broadband operation (i.e., with negligible frequency distortion over the essential bandwidth of the input signal) around the centre frequency when the (actual) interaction length is chosen to be $\bar{d} = v_0 \, d_{opt} / 2\pi \, f_c$.

FUTURE SCOPE

Work reported in this thesis may be extended in various possible directions.

The sheath-helix model of the slow-wave structure is inconsistent with any arrangement for coupling power into and out of the TWTA. In practice, the two ends of a round-wire helix are extended to form the external conductors of pair of coaxial lines through which r. f. power is coupled in and out of the tube. Unfortunately, the sheath-helix model of the slow-wave structure is incompatible with this coupling arrangement. For this reason it is impossible to rigorously account for the perturbation of the field configuration resulting from the input and the output connections within the framework of the sheath-helix model. In this context, it will be of interest to model the slow-wave structure to be a finite-length tape helix which is indeed compatible with input and output coupling arrangements employing shielded strip lines to carry microwave power into and out of the TWTA.

Further, the present analysis has made use of an infinite focusing magnetic field giving rise to a fully confined electron beam, partially filling the tube. This analysis may be extended to a more realistic (practical) case of a finite focussing field with a nonzero radial component. With finite focussing, the electron trajectories become three-dimensional, giving rise to a beam boundary which varies with the axial position and the time. For a fixed axial position, the time variation of beam boundary will be periodic with a period equal to the reciprocal of the fundamental operating frequency of the tube. The feasibility of a large-signal field analysis of a TWTA with a finite focusing magnetic field and an open sheath-helix model for the slow-wave structure has already been established [10].

ORGANIZATION OF THESIS

This thesis presents a field-theoretic analysis of the nonlinear electron-wave interaction taking place in a linear-beam traveling wave amplifier operating in an axially-symmetric mode with an axially confined electron beam.

The thesis is organized in the following way.

Chapter 1 introduces the importance of the traveling wave tube amplifiers and its adequacy in the high power applications like military and radar applications and satellite communication. Chapter 1 also describes the types of traveling wave tube amplifiers. Brief introduction about the construction and working principle of TWTAs are provided. Work by the previous researchers in early 1900s with the invention of TWTA by Rudolf Komphner to the work carried out till on TWTAs are extensively reviewed in the literature survey section of Chapter 1.

Chapter 2 follows with the problem formulation of the field problem and brief explanation about the various simplifying assumptions, notations, terminology and the helix slow wave structure are provided in the initial sections of this chapter. The description of the model used in the analysis and the boundary conditions are dealt in detail. Also, the Maxwell's equations, electron ballistic equation and the current density are briefly discussed.

Chapter 3 introduces the solution procedure of the field problem. The key step in the analysis is the representation of the field components as nonlinear functionals of the electron arrival time through a Green's function sequence for the slow-wave circuit. Homogeneous solution and the particular solutions for the field components are obtained. The axial electric field component is derived in terms of electron arrival time. Substitution of this functional representation for the axial electric field component into the electron ballistic equation casts the latter into a fixed point format for a nonlinear operator in an appropriate function space. The fixed point, and therefore the solution for the electron-arrival time and hence the solution for the electromagnetic field components, can be obtained by standard successive approximation techniques.

Chapter 4 describes the modifications required in the particular solution for the field components, when there is resonance. This chapter deals with a special case when the interaction length of the traveling wave amplifier is an integral multiple of a cold wavelength at the m^{th} harmonic of the input signal frequency. This is known as the resonance case.

Chapter 5 is concluded by the numerical scheme and a discussion of results. A numerical solution for the electron arrival time is obtained for a set of typical parameter values. The calculations of the gain, the efficiency, surface current density and the other amplifier parameters, comparison of the results of the present theory with experimental results on the basis of a successive approximation solution for the field components will be discussed.

Chapter 6 discusses the conclusions arrived at field analysis of the dielectric-loaded TWTA. Future directions of research in this field are also discussed in this chapter.

Appendix A gives a proof of the change-of-variables formula used to express the Fourier coefficients of charge and current densities in terms of electron arrival time. Appendix B gives a proof of contraction mapping theorem.

PUBLICATIONS

1. **Kalyanasundaram, N.** and **Agnihotri A.**, "Large signal field analysis of linear beam traveling wave amplifier for a sheath-helix model of the slow-wave structure supported by dielectric rods. Part 1: Theory," Progress in Electromagnetic Research *B*, vol. 57, pp. 87-104, 2014 [Impact factor : 5.298, h5-Index: 25, h5-Median: 37].

2. **Kalyanasundaram, N.** and **Agnihotri A.**, "Large signal field analysis of linear beam traveling wave amplifier for a sheath-helix model of the slow-wave structure supported by dielectric rods. Part 2: Numerical results," Progress in Electromagnetic Research *B*, vol. 57, pp. 105-114, 2014 [Impact factor : 5.298, h5-Index: 25, h5-Median: 37].

REFERENCES

1. Pierce, J. R. and L. M. Field, *"Traveling wave tubes,"* Proc. IRE, Vol. 35, 108-111, 1947.
2. Pierce, J. R., *"Theory of the beam-type traveling wave tube," Proc. IRE*, Vol. 35, 111-123, 1947.
3. Komfner, R., *"The Traveling wave tube as amplifier at microwaves," Proc. IRE*, Vol. 35, 124-128, 1947.
4. Rydbeck, O. E. H., *"Theory of the traveling wave tubes," Ericsson Technics*, Vol. 46, 03-18, 1948.

5. Chu, L. J. and J. D. Jackson, *"Field theory of traveling wave tubes," Proc. IRE*, Vol. 36, 853-863, 1948.
6. Collin, R. E., *Foundations for Microwave Engineering*, 2nd edition, IEEE Press, 2005.
7. Freund, H. P., M. A. Kodis and N. R. Vanderplaats, *"Design of traveling wave tubes based on field Theory,"* IEEE Trans. Electron Devices, Vol. 41, No. 7, 1288-1296, 1994.
8. Freund, H. P., E. G. Zaidman, M. A. Kodis and N. R. Vanderplaats, *"Linearized field theory of a dielectric-loaded helix traveling wave tube amplifier,"* IEEE Trans. Plasma Science, Vol. 24, No. 3, 895-904, 1996.
9. Kalyanasundaram, N., *"Large signal field analysis of an O-type traveling wave amplifier. Part 1: Theory,"* IEE Proc. I, Solid-State &Electron Dev., Vol. 131, No. 5, 145-152, 1984.
10. Kalyanasundaram, N. and R. Chinnadurai, *"Large signal field analysis of an O-type traveling wave amplifier. Part 3: Three-dimensional electron motion," IEE Proc. I,* Vol. 135, No. 3, 59-66, 1988.
11. Kalyanasundaram, N. and R. Chinnadurai, *"Large signal field analysis of an O-type traveling wave amplifier. Part 2: Numerical results,"* IEE Proc. I, Solid-State & Electron Dev., Vol. 133, No. 4, 163-168, 1986.
12. Rowe, J. E., *Nonlinear Electron-Wave Interaction Phenomena*, Academic Press, 1965.
13. Detweiler, H. K. and J. E. Rowe, *"Electron dynamics and energy conversion in O-type linear beam devices,"* in Young, L.(Ed.), *Advances in Microwaves 6,* Academic Press, 29-123, 1971.
14. Hughes Aircraft Company, Electron Dynamics Division, *TWT and TWTA Handbook*, Torrance, California, 1975.
15. Gewartowski, J.W. and H.A. Watson, *Principles of Electron Tubes*, Van Nostrand, 1968.

16. Jain, P. K. and B. N. Basu, *"The Inhomogeneous loading effects of practical dielectric supports for the helical slow-wave structure of a TWT, "* IEEE Trans. Electron Devices, Vol. 34, No. 12, 2643-2648, 1987.
17. Naylor A.W. and G.R. Sell, *Linear Operator Theory in Engineering and Science,* Springer-Verlag, 1982.
18. Thomas, J.B., *Introduction to Probability*, Springer-Verlag, 1986.
19. Jacod, J. and P. Protter, *Probability Essentials*, 2nd edition, Springer, 2003.
20. Kalyanasundaram, N. and G. N. Babu, *"Propagation of electromagnetic waves guided by anisotropically conducting model of a tape helix supported by dielectric rods,"* Progress In Electromagnetic Research B, Vol. 51, 81-99, 2013.
21. P. Birtel et al., *"Simulation of the BWO threshold current in a helix TWT,"* IEEE Trans. IVEC, pp. 403-404, 2006.
22. R. Harper, and M. P. Puri, *"Heat transfer and power capabilities of EHF helix TWTs," Technical Digest, IEDM*, pp. 498-500, 1986.
23. NASA Lewis Research Centre, *"Coupled-cavity traveling-wave tube has phase-adjusted taper," NASA Tech Briefs*, pp. 26-30, 1992.
24. Shulim E. Tsimring, *Electron Beams and Microwave Vacuum Electronics,* John Wiley & Sons. Inc., 2007.
25. Keqian Zhang and Dejie Li, *Electromagnetic Theory for Microwaves and Optoelectronics*, Springer Berlin Heidelberg New York, 1998.
26. Ishli, T. K., The Electronics Handbook, Whitaker, J. C., Ed., CRC Press, Boca Raton, FL, 1996.