NON-LINEAR PROBLEMS IN MICROPOLAR FLUID FLOWS

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By

DIKSHA GUPTA

विद्या तत्व ज्योतिसम:

Department of Mathematics

JAYPEE INSTITUTE OF INFORMATION TECHNOLOGY
(Declared Deemed to be University U/S 3 of UGC Act)
A-10, SECTOR-62, NOIDA, INDIA

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SYNOPSIS

INTRODUCTION

We live in an environment of fluids. The air we breathe and the water we drink are fluids. Motion of air keeps us comfortable and provides the oxygen we need to sustain life. Specifically, a fluid is defined as a substance that deforms continuously when acted on by a shearing stress of any magnitude. Thus, fluids comprise the liquid and gas phases of the physical forms in which matter exists. Fluids can be characterized as Newtonian and non-Newtonian fluids. For Newtonian fluids the shearing stress is linearly related to the rate of shearing strain but for non-Newtonian fluids shearing stress is not linearly related to the rate of shearing strain. Water, air, mercury, kerosene and thin lubricating oils are Newtonian fluids whereas paints, coal tar, blood and grease represent for non-Newtonian fluids.

The behaviour of a fluid flow can be characterized by a set of partial differential equations, these equations are based on the conservation of mass, linear momentum and energy. Fluid mechanics is the discipline in which we study the behaviour of fluids, both at rest or in motion. Fluid dynamics concerns itself with the study of the motion of the fluids. Like the study of any other branch of science, fluid dynamics needs mathematical analysis as well as experimentation. Mathematical approach to study the behaviour of fluids using computer, is undertaken under the field computational fluid dynamics, while fluid dynamics via experimentation is studied under disciplines like hydraulics and aeronautical engineering.

The ultimate goal of computational fluid dynamics is the analysis of systems involving fluid flow, heat transfer and associated phenomena such as diffusion, convection, dissipation, boundary layers and turbulence by means of computer based simulation. All of these phenomena are governed by the compressible Navier-Stokes equations. Many of the important aspects of these relations are non-linear and as a result, often have no analytical solution. In order to obtain the solution of the associated partial differential equations one must therefore apply numerical methods. Computational fluid dynamics is the art and science of computing such solutions. It encompasses both development and application of the numerical methods using computer coding.

The Navier-Stokes model of classical hydrodynamics has a major limitation. It cannot describe fluids with microstructure like polymeric suspensions, animal blood, body fluids, liquid crystals and lubricating oils etc. These fluids are interesting in themselves and significant in applications. Generally, individual particles of these fluids can be of different shape, they can expand and shrink, can rotate independently of the rotation and movement of the fluid. In order to define the behaviour of these fluids accurately, a theory is required that takes into consideration geometry, deformation and intrinsic motion of individual material particles. Many such theories have appeared in continuum mechanics, some of them are theory of simple microfluids, micropolar fluids, simple deformable directed fluids and dipolar fluids.

Eringen [1] introduced the theory of simple microfluids. According to this theory the simple microfluid is a fluent medium whose properties and behaviour are affected by the local motion of the fluid elements. These fluids can support body moments and stress moments and are influenced by the spin inertia. The stress tensor for these fluids is non-symmetric. This theory has presented an excellent model to examine many complex fluids some of them are liquid crystals, the flow of low concentration suspensions, blood and turbulent shear flows. However, a severe problem is encountered when this theory is applied to real nontrivial flow problems; even for the linear theory, a problem contains system of 19 partial differential equations with 19 unknowns. This makes the model very complex and the mathematical model is not so acceptable to the solution of the non-trivial problems.

Keeping this in mind, Eringen [2] simplified the theory of microfluids and defined a subclass of these fluids called micropolar fluids. This theory is one of the best established theories of fluids with microstructure. Micropolar fluids represent fluids consisting of rigid randomly oriented particles suspended in a viscous medium, where the deformation of particles is ignored. The attractiveness and power of the model of micropolar fluids lie in the fact that it is both a significant and a simple generalization of the classical Navier-Stokes model. According to classical Navier-Stokes model each material particle possess three translational degrees of freedom whereas micropolar fluids possess six degrees of freedom, three translational and three rotational degrees of freedom. The rotational degrees of freedom bring into play nonsymmetrical stress tensor and couple stresses, which were missing from the classical theory. Physically micropolar fluids represent the fluids consisting of bar-like elements and certain anisotropic fluids, for example, animal blood, liquid crystals which are made up of dumpbell molecules.

In the theory of micropolar fluid, two new variables of the velocity are added which were not presented in the Navier-Stokes model. These are micro-rotations variables that represent spin and micro-inertia tensors which describe the distribution of atoms and molecules inside the microscopic fluid particles. This theory has a significant feature that the microrotation bears a resemblance to the vorticity. Only those components of microrotation are non-zero which corresponds to the non-vanishing components of vorticity and their dependence is very analogous to as that of vorticity. In this theory, four new viscosities are presented. Out of these four viscosities if the microrotation viscosity becomes zero, the law of conservation of linear momentum becomes independent of the presence of the microstructure. Hence the size of microrotation viscosity coefficient enables us to measure, the deviation of flows of micropolar fluids from that of Navier-Stokes model. The detailed theory of micropolar fluids can be found in the books by Eringen [3] and Lukaszewicz [4].

Micropolar fluids have found a large number of applications in the description of physical phenomena which cannot be well described by classical fluid mechanics. Among these are liquid crystals and occurrence of turbulence. The other applications include lubrication problem, Stokes flow about a sphere, stagnation flow, Taylor-Benard instability, boundary layer flow over a plate. Also body fluids and biological flow problems have been modeled by micropolar theory. These fluids reduce the skin friction as shown experimentally by Hoyat and Fabula [5] and Voge and Patterson [6]. The most comprehensive discussion of applications can be found in the review article of Ariman *et al.* [7]. In this article extensive individual flow cases have been summarized. Micropolar fluid lubrication with reference to human joints is discussed in the research papers by Nigam *et al.* [8] and Sinha *et al.* [9]. Numerous applications [10] of micropolar fluids abound in industrial engineering, geophysics, energy systems, biomechanics etc.

The boundary layer theory was presented by Ludwig Prandtl. The main idea was to divide the flow into two parts. The smaller part is a thin layer in the vicinity of solid surface in which the effects of viscosity are felt. This thin layer near the solid surface is called boundary layer. The larger part concerns a free stream of fluid, far from solid surface, which is considered to be non-viscous. Although the boundary layer is thin, it plays an essential role in fluid dynamics. It has become an excellent method for analyzing the complex behaviour of real fluids. The concept of boundary layer can be used to simplify the Navier-Stokes equations to such an extent that it becomes possible to tackle a large number of practical problems of great importance. The boundary layer theory is used very frequently for solving fluid flow and heat transfer problems.

In manufacturing processes, the boundary layer flow induced by stretching surface has various theoretical and technical applications. Some of these applications include wire drawing, paper production, glass-fiber production, liquid metal, polymer sheet synthesis, continuous stretching of plastic films and artificial fibers etc. In all these cases, the mechanical properties of the final product are appreciably affected by the rate of cooling and stretching in the process and material characteristics could therefore be manipulated to desired specifications. The pioneering work in this area was carried out by Sakiadis [11]. Peddieson and McNitt [12] investigated the boundary layer flow of micropolar fluid. Various aspects of micropolar fluid flows from a stretching surface have been studied by many authors [13-19].

Recently, the boundary layer flow of an incompressible fluid over a shrinking sheet has attracted the interest of researchers due to its applications in polymeric materials processing. Flow due to a shrinking sheet is different from the stretching sheet flow. A stretching sheet regime induces far field suction towards the sheet, while a shrinking sheet intensifies the velocity away from the sheet. Shrinking sheet flows are important in the manufacture of certain polymers and high-performance materials for aerospace coatings (Baird and Baird [20]), processing of various non-Newtonian materials for ceramic suspensions (Zhong *et al.* [21]), thermal shrinking of polythene sheets (Gupta and Ward [22]), viscoelastic membranes used in petroleum applications (Cheremisinoff [23]). Many theoretical and numerical studies of such flows have been reported by many researchers [24-29].

The convective heat transfer is mainly divided into two basic processes, free convection and forced convection. In several practical applications, the temperature difference exists in the boundary layer region near heated or cooled surface. Due to temperature difference density gradients appear in the fluid medium and in the presence of gravitational force free convection effects arise. These density differences give rise to buoyancy forces. If the motion of the fluid arises from an external agent, then the process is called forced convection. Thus in any forced convection situation, free convection effects are also present. If the effect of forced flow in free convection or the effect of buoyancy force in free convection becomes significant, then this process is called mixed convection. Mixed convection flows arise frequently in process mechanical engineering and chemical processing. Interesting studies of mixed convection flow of a micropolar fluid have been reported by many researchers [30-34].

Magnetohydrodynamics is the branch of continuum mechanics which deals with the motion of an electrically conducting fluid in the presence of a magnetic field. Many technological problems and natural phenomena are susceptible to MHD analysis. Engineers apply MHD principle, in the design of heat exchangers, in creating novel power generating systems, pumps and flow meters, thermal protection, braking, control and re-entry, in space vehicle propulsion. MHD convection flow problems are also very important in the fields of stellar and planetary magnetospheres, aeronautics, electronics and chemical engineering. Many theoretical studies of magneto-micropolar fluids have been reported by different researchers [35-41].

The involvement and applications of mass transfer process go to a greater length in multiple fields of science, engineering and technology. The transport of one component in a mixture from a region of high concentration to a region of low concentration is called mass transfer. Mass transfer operations quite often occur in industrial, biological, physical and chemical engineering processes. It is used by different scientific disciplines for different processes and mechanisms. Coupled heat and mass transfer flows constitute a major area of research in modern fluid dynamics. Such flows arise in electronic cooling, drying processes manufacture of electric cable insulations, curing of plastics, solar energy system and purification processes. Many researchers [42-48] have investigated various problems of heat and mass transfer in micropolar fluids including different features.

In the present thesis we have studied various problems of micropolar fluid flows in different geometries. The equations governing the flow of micropolar fluids are non-linear in nature and cannot be solved analytically. Therefore, the only option for solving such problems is the numerical approach. Various numerical methods such as Quasi-linearization, Finite element method, Finite difference method, Shooting method, Keller-box method etc. can be used for solving these problems. In the present work we have used Quasi-linearization [49] and Finite element method [50].

OBJECTIVES

The main objectives of the work reported in the thesis are to:

- obtain mathematical model for the various non-linear problems of micropolar fluid flows.
- find the numerical solution of these problems.
- study the effect of various parameters on the flow and heat transfer characteristics of micropolar fluid.
- find the parameters responsible for the reduction of skin friction.
- identify the parameters which increase the rate of heat transfer.

MAIN CONTRIBUTION

The work embodied in the present thesis is divided into nine chapters. We have considered seven problems out of which five are for steady flow and two are for unsteady flow. Six problems are solved using finite element method whereas one problem is solved using Quasi-linearization technique along with Runge-Kutta method. In all these problems under special conditions, the results obtained numerically are compared either with the exact solution or with the numerical results available from the literature. Excellent agreement between the results is obtained. Also, the convergence of the numerical solutions is observed as the numbers of elements is increased. The influence of various important physical parameters on velocity, microrotation and temperature distribution is studied extensively and is depicted graphically. The skin-friction coefficient, local couple stress and the local Nusselt number have also been computed. The chapter wise summary of the work done is given as follows.

Chapter 1: This chapter is introductory and sets up the background for the problems taken in the thesis. It reviews existing literature relevant to the thesis e.g. micropolar fluid, their basic equations and applications, boundary layer theory, applications of stretching sheet, heat and mass transfer, magnetohydrodynamics and work done on micropolar fluids. Additionally a concise description of the Quasi-linearization and finite element method is also included.

Chapter 2: In this chapter a mathematical model is developed to study the effect of thermal radiation on mixed convection flow of micropolar fluid over a shrinking sheet with prescribed heat flux. The velocity of the shrinking sheet and the surface heat flux are assumed to vary as a linear function of the distance from the origin. The results are presented graphically for velocity, microrotation and temperature functions with various values of physical parameters such as suction, radiation and buoyancy parameters.

Chapter 3: This chapter describes the steady mixed convection magnetohydrodynamic flow of an electrically-conducting micropolar fluid over a porous shrinking sheet. The velocity of shrinking sheet and magnetic field are assumed to vary as a power function, of the distance from the origin. A convective boundary condition is used rather than the customary conditions for temperature i.e. constant surface temperature or constant heat flux. The influence of various emerging thermophysical parameters, namely suction parameter, convective heat transfer parameter, magnetic parameter and power index on velocity, microrotation and temperature functions is studied extensively and shown graphically.

Chapter 4: In this chapter we investigate theoretically and computationally, the stagnation-point flow and heat transfer of an electrically-conducting micropolar fluid from a stretching/shrinking sheet in the presence of melting and viscous heating. The stretching/shrinking and the ambient fluid velocities are assumed to vary linearly with the distance from the stagnation-point. The influence of magnetic, stretching/shrinking and melting parameters is thoroughly examined over the velocity, microrotation and temperature distributions.

Chapter 5: In this chapter we discuss the mechanical and thermal characteristics of boundary layer stagnation-point flow of MHD micropolar fluid over a porous stretching sheet. An isothermal surface stretched with constant skin friction is considered. A uniform magnetic field is applied perpendicular to the porous stretching sheet. The influence of the key physical parameters namely, buoyancy parameter, magnetic parameter and transpiration parameter on the evolution of velocity, microrotation and temperature functions is presented graphically.

Chapter 6: The objective of this chapter is to analyze theoretically and numerically the effect of thermal radiation and viscous dissipation on mixed convection flow of micropolar fluid from a continuously moving vertical porous sheet. Detailed computations are presented for velocity, microrotation and temperature functions and a parametric study of the influence of wall suction (or injection), buoyancy parameter, viscous heating and radiation parameters is conducted.

Chapter 7: This chapter presents the unsteady mixed convection boundary layer flow of micropolar fluid over a permeable shrinking sheet in the presence of viscous dissipation. At the sheet a variable distribution of suction is assumed. The unsteadiness in the flow and temperature fields is caused by the time dependence of the shrinking velocity and surface temperature. The influence of important physical parameters namely, suction parameter, unsteadiness parameter, buoyancy parameter and Eckert number on the velocity, microrotation and temperature functions are investigated and analyzed with the help of their graphical representations.

Chapter 8: In this chapter a mathematical model is developed to study the unsteady laminar heat and mass transfer in the incompressible micropolar boundary layer flow from a porous stretching sheet with variable suction. The unsteadiness in the flow, temperature and concentration fields is caused by the time-dependence of the stretching velocity, surface temperature and surface concentration. The effect of suction parameter, unsteadiness parameter, coupling constant parameter and Schmidt number on the velocity, microrotation, temperature and concentration functions is examined.

Chapter 9: It is the final and concluding chapter, which contains the summary and concluding remarks of the thesis as well as some suggestions for future scope. This chapter is followed by the bibliography.

LIST OF PUBLICATIONS

International Journals

Published Papers

1. Gupta D., Kumar L., Bég O.A., Singh B., "*Finite element simulation of mixed convection flow of micropolar fluid over a shrinking sheet with thermal radiation*", Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering, vol. 228, pp. 61-72, 2014.
   Indexed in: Science Citation Index, Scopus, **H Index:** 16, **Impact Factor:** 0.547
2. Gupta D., Kumar L., Singh B., "*Finite element solution of unsteady mixed convection flow of micropolar fluid over a porous shrinking sheet*", The Scientific World Journal, vol. 2014, pp. 11, Article ID 362351, 2014.
   Indexed in: Science Citation Index Expended, Scopus, **H Index:** 40,
   Impact Factor: 1.219
3. Gupta D., Kumar L., Bég O.A., Singh B., "*Finite element analysis of transient heat and mass transfer in microstructural boundary layer flow from a porous stretching sheet*", Computational Thermal Sciences, vol. 6, pp. 155-169, 2014. **Indexed in:** Scopus, **H Index:** 7

Communicated Papers

1. Gupta D., Kumar L., Bég O.A., Singh B., "*Finite element simulation of nonlinear magneto-micropolar stagnation point flow from a porous stretching sheet with prescribed skin friction*", communicated to Computational Thermal Sciences (June 2014).
   Indexed in: Scopus, **H Index:** 7
2. Gupta D., Kumar L., Bég O.A., Singh B., "*Numerical study of steady dissipative mixed convection optically-thick micropolar flow with thermal radiation effects*", communicated to Journal of Applied Fluid Mechanics (June 2014).
   Indexed in: Science Citation Index Expended, Scopus, **H Index:** 8,
   Impact Factor: 0.505
3. Gupta D., Kumar L., Singh B., Bég O.A., "*Finite element analysis of melting effect on MHD stagnation-point non-Newtonian flow and heat transfer from a*

stretching/shrinking sheet", communicated to International Journal of Energy & Technology (June 2014).

4. Gupta D., Kumar L., Bég O.A., Singh B., *"Finite element solution of MHD flow of micropolar fluid over a shrinking sheet with a convective surface boundary condition*", communicated to Journal of Engineering Thermophysics (June 2014).
   Indexed in: Science Citation Index Expended, Scopus, **H Index:** 7,
   Impact Factor: 0.522

Papers presented in International/National Conferences

1. Gupta D., Kumar L., Singh B., "Numerical solution of MHD flow of micropolar fluid over a non-linear shrinking sheet with a convective boundary condition" Book of abstract pp. 19, National Conference on Contemporary Developments in Mathematical Sciences and Computing (CDMSC-2013) February 2-3, 2013, Department of Mathematics, Galgotias University, Greater Noida, Uttar Pradesh.
2. Gupta D., Kumar L., Singh B., "*Stagnation-point flow of micropolar fluid over a stretching/shrinking sheet with melting and radiation effects*" Book of abstract pp. 20, National Conference on Modeling, Computational Fluid Dynamics and Operations Research (NCMCO-2012) February 4-5, 2012, Department of Mathematics, BITS Pilani, Pilani Campus, Rajasthan.
3. Gupta D., Kumar L., Singh B., "*Numerical solution of the effect of thermal radiation on the flow of micropolar fluid over a shrinking sheet*", Book of abstract pp. 15, International Conference on Advances in Modeling, Optimization and Computing (AMOC -2011) December 5-7, 2011, Department of Mathematics, IIT Roorkee.
4. Gupta D., Kumar L., Singh B., "*Numerical solution of the mixed convection micropolar fluid past a continuously moving plate in the presence of radiation*", Book of abstract pp. 26, National Symposium on Application of Various Techniques in Fluid Dynamics (NSAVTFD-2011), February 10-12, 2011, Department of Mathematics, B.S.N.V. Post Graduate College, Lucknow in Association with National Science Network (NSN).

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Dr. Lokendra Kumar	Prof. Bani Singh	Diksha Gupta
(Supervisor)	(Supervisor)	(Research Scholar)